The Price:Earnings Ratio (a.k.a PE ratio)
The Greeks And Insights

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In mathematical finance, the Greeks are the quantities (known in calculus as partial derivatives; first-order or higher) representing the sensitivity of the price of an asset to changes in one or more underlying parameters on which the value that asset is dependent. In this white paper we will calculate the derivatives of share price. To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

We are tasked with estimating the sensitivity of share value of ABC Company given a change in one or more model parameters. The following table presents our model assumptions...

Description	Value
Operating assets ($)	1,000,000
Number of common shares (#)	100,000
Cost of capital (%)	9.52
Assets growth rate (%)	4.87
Economic profits (%)	1.50

Question 1: What is market value and book value per share?

Question 2: What is the equation for share price if the economic profits rate is zero?

Question 3: What is the change in share price if the economic profits rate increases by 50 bps?

Question 4: What is the change in share price if the earnings growth rat increases by 75 bps?

Note: To simplify the model, we will assume that balance sheet assets are funded 100% with common equity capital. This means that the after-tax return on assets and the after-tax return on capital are the same. Economic profits is the rate of return on assets over and above the cost of capital.

Building Our Model

We defined the variable S_t to be share price at time t, the variable A_t to be operating assets at time t, the variable N_t to be the number of fully-diluted shares at time t, the variable π to be the after-tax return on assets, the variable μ to be the continuous-time earnings growth rate, and the variable κ to be the continuous-time cost of capital (i.e. discount rate).

$$S_0 = \pi \, \frac{A_0}{N_0} \, \frac{1 - \mu/\pi}{\kappa - \mu} \tag{1}$$

We will define operating assets to be total GAAP assets cash and cash equivalents, goodwill, and other non-operating assets. Using this definition and Equation (1) above, we will make the followin observation...

$$\frac{A_0}{N_0} = \text{Adjusted book value per share} \tag{2}$$

We defined the variable ω_t to be annualized earnings per share at time t and the variable θ to be the price-to-earnings ratio. Using Equation (1) above, the equations for earnings per share at time zero and the PE ratio are...

$$\omega_0 = \text{Earnings per share} = \pi \, \frac{A_0}{N_0} \;\; \text{...and...} \;\; \theta = \text{PE ratio} = \left(1 - \frac{\mu}{\pi}\right)\left(\kappa - \mu\right)^{-1} \tag{3}$$

We defined the variable λ to be economic profits, which is the rate of return that the company earns over and above the company's cost of capital. Using this definition, the equation for the company's return on assets (π) is...

$$\pi = \text{After-tax return on assets} = \kappa + \lambda \tag{4}$$

Using Equation (4) above, we can rewrite Equation (3) above as...

$$\omega_0 = \text{Earnings per share} = (\kappa + \lambda) \, \frac{A_0}{N_0} \;\; \text{...and...} \;\; \theta = \text{PE ratio} = \left(1 - \frac{\mu}{\kappa + \lambda}\right)\left(\kappa - \mu\right)^{-1} \tag{5}$$

We define the variable S_t to be share price at time t. Uisng Equation (5) above, the equation for share price at time zero is...

$$S_0 = \omega_0 \, \theta \tag{6}$$

Using Appendix Equation (17) below, the solution to Equation (6) is...

$$S_0 = \frac{A_0}{N_0} \, \frac{\kappa + \lambda - \mu}{\kappa - \mu} \tag{7}$$

Note that we can break Equation (7) above into two parts...

$$\frac{A_0}{N_0} = \text{Adjusted book value per share} \;\; \text{...and...} \;\; \frac{\kappa + \lambda - \mu}{\kappa - \mu} = \text{Valuation multiple of adjusted book value} \tag{8}$$

Using Appendix Equations (18) and (20) below, the first derivatives of the valuation multiple in Equation (8) above with respect to excess return variable λ and growth rate variable μ are...

$$\frac{\delta}{\delta \lambda} \, \frac{\kappa + \lambda - \mu}{\kappa - \mu} = \frac{1}{\kappa - \mu} \;\; \text{...and...} \;\; \frac{\delta}{\delta \mu} \, \frac{\kappa + \lambda - \mu}{\kappa - \mu} = \frac{\lambda}{(\kappa - \mu)^2} \tag{9}$$

Using Appendix Equations (19) and (21) below, the second derivatives of the valuation multiple in Equation (8) above with respect to excess return variable λ and growth rate variable μ are...

$$\frac{\delta^2}{\delta \lambda^2} \, \frac{\kappa + \lambda - \mu}{\kappa - \mu} = 0 \;\; \text{...and...} \;\; \frac{\delta^2}{\delta \mu^2} \, \frac{\kappa + \lambda - \mu}{\kappa - \mu} = \frac{2\,\lambda}{(\kappa - \mu)^3} \tag{10}$$

Using Equations (7), (9) and (10) above, the equation for the change in share price via a Taylor Series Expansion is...

$$\delta S_0 = \frac{A_0}{N_0} \left[\frac{1}{\kappa - \mu} \, \delta \lambda + \frac{\lambda}{(\kappa - \mu)^2} \delta \mu + \frac{1}{2} \, \frac{2\,\lambda}{(\kappa - \mu)^3} \delta \mu^2 \right] \tag{11}$$

Answers To Our Hypothetical Problem

Question 1: What is market and book value per share?

Using Equation (2) above and the model parameters in the table above, book value per share is...

$$\text{Book value per share} = \frac{A_0}{N_0} = \frac{\$1,000,000}{100,000} = \$10.00 \tag{12}$$

Using Equations (7) and (12) above and the model parameters in the table above, market value per share is...

$$S_0 = \$10.00 \times \frac{0.0952 + 0.0150 - 0.0487}{0.0952 - 0.0487} = \$13.23 \tag{13}$$

Question 2: What is the equation for share price if the economic profits rate is zero? Using Equations (7) and (12) above and the model parameters in the table above, the answer to the question is...

$$\text{if... } \lambda = 0 \text{ ...then... } S_0 = \frac{A_0}{N_0} \frac{\kappa + 0 - \mu}{\kappa - \mu} = \frac{A_0}{N_0} = \text{Book value per share} = \$10.00 \tag{14}$$

Insight 01 = If after-tax return on assets equals the cost of capital then share price equals book value per share.

Insight 02 = If after-tax return on assets equals the cost of capital then growth rate μ is irrelevant.

Question 3: What is the change in share price if the economic profits rate increases by 50 bps?

Using Equation (11) above and the model parameters in the table above, the answer to the question is...

$$\text{Change in share price} = \frac{1,000,000}{100,000} \times \frac{1}{0.0952 - 0.0487} \times 0.0050 = \$1.08 \tag{15}$$

Insight 03 = Increase the economic profits rate and share price increases.

Question 4: What is the change in share price if the earnings growth rat increases by 75 bps?

Using Equation (11) above and the model parameters in the table above, the answer to the question is...

$$\text{Change in share price} = \frac{1,000,000}{100,000} \times \left(\frac{0.0150}{0.0952 - 0.0487} + \frac{1}{2} \times \frac{2 \times 0.150}{(0.0952 - .0487)^2} \right) \times 0.0050 = \$0.63 \tag{16}$$

Insight 04 = Increase the growth rate and share price increases.

Appendix

A. Using Equation (3) and (4) above, book value per share is...

$$\begin{aligned} S_0 = \omega_0 \, \theta &= (\kappa + \lambda) \frac{A_0}{N_0} \left(1 - \frac{\mu}{\kappa + \lambda} \right) \left(\kappa - \mu \right)^{-1} \\ &= (\kappa + \lambda) \frac{A_0}{N_0} \left(\frac{\kappa + \lambda - \mu}{\kappa + \lambda} \right) \left(\kappa - \mu \right)^{-1} \\ &= \frac{A_0}{N_0} \frac{\kappa + \lambda - \mu}{\kappa - \mu} \end{aligned} \tag{17}$$

Using Equation (3) and (4) above, book value per share is...

B. The first derivative of the following equation with respect to λ via the quotient rule is...

$$\begin{aligned} \frac{\delta}{\delta \lambda} \frac{\kappa + \lambda - \mu}{\kappa - \mu} &= \left(1 \times (\kappa - \mu) - 0 \times \kappa + \lambda - \mu \right) \left(\kappa - \mu \right)^{-2} \\ &= \left(\kappa - \mu \right) \left(\kappa - \mu \right)^{-2} \\ &= \frac{1}{\kappa - \mu} \end{aligned} \tag{18}$$

The second derivative of Equation (18) above with respect to λ via the quotient rule is...

$$\begin{aligned} \frac{\delta^2}{\delta \lambda^2} \frac{\kappa + \lambda - \mu}{\kappa - \mu} &= \frac{\delta}{\delta \lambda} \frac{1}{\kappa - \mu} \\ &= \left(0 \times (\kappa - \mu) - 0 \times 1 \right) \left(\kappa - \mu \right)^{2} \\ &= 0 \end{aligned} \tag{19}$$

C. The derivative of the following equation with respect to μ via the quotient rule is...

$$\frac{\delta}{\delta\mu}\frac{\kappa+\lambda-\mu}{\kappa-\mu} = \Bigg(-1\times(\kappa-\mu) - -1\times\kappa+\lambda-\mu\Bigg)\Bigg(\kappa-\mu\Bigg)^{-2}$$
$$= \Bigg(\mu-\kappa+\kappa+\lambda-\mu\Bigg)\Bigg(\kappa-\mu\Bigg)^{-2}$$
$$= \frac{\lambda}{(\kappa-\mu)^2} \tag{20}$$

The second derivative of Equation (20) above with respect to μ via the quotient rule is...

$$\frac{\delta^2}{\delta\mu^2}\frac{\kappa+\lambda-\mu}{\kappa-\mu} = \frac{\delta}{\delta\mu}\frac{\lambda}{(\kappa-\mu)^2}$$
$$= \Bigg(2\times(\kappa-\mu)\times\lambda - 0\times(\kappa-\mu)^2\Bigg)\Bigg(\kappa-\mu\Bigg)^{4}$$
$$= \Bigg(2\,\lambda\,(\kappa-\mu)\Bigg)\Bigg(\kappa-\mu\Bigg)^{4}$$
$$= \frac{2\,\lambda}{(\kappa-\mu)^3} \tag{21}$$